

# Entanglement and quantum teleportation via decohered tripartite entangled states

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## Abstract

The entanglement behavior of two classes of multi-qubit system, GHZ and GHZ like states passing through a generalized amplitude damping channel is discussed. Despite this channel causes degradation of the entangled properties and consequently their abilities to perform quantum teleportation, one can always improve the lower values of the entanglement and the fidelity of the teleported state by controlling on Bell measurements, analyzer angle and channel's strength. Using GHZ-like state within a generalized amplitude damping channel is much better than using the normal GHZ-state, where the decay rate of entanglement and the fidelity of the teleported states are smaller than those depicted for GHZ state.

## 1 Introduction

Decoherence represents one of the inevitable phenomena in quantum information tasks. It is possible to generate maximum entangled state, but keeping it isolated from its surrounding is a big challenge. Therefore, sometimes we are forced to use these decohered entangled states to achieve some quantum information tasks. So, investigating the entangled properties of initial entangled states passing through noise channels are of a great importance.

Practically, it is often required to send some parts of the generated entangled states from the source to remote users [1]. Each transmitted subsystem interacts with its environment locally and consequently leads to lose some of entangled properties of the multipartite system [2, 3, 4, 5]. Consequently, the efficiency of using these decohered entangled states as quantum channels to perform quantum teleportation decreases. However, there are some efforts have been introduced to recover and protect entanglement from degradation [6, 7]. Despite this degradation, it has been shown that local environment can enhance the fidelity of quantum teleportation [8, 9].

The dynamics of a single and two qubit states in noisy channel has been studied extensively. The most important channels are phase flip, depolarize and amplitude damping channels [10, 11]. Recently, Dontealegre et. al have shown that the effect of the generalized amplitude damping channel can be frozen [4, 12]. Metwally [5] has shown that under the effect of the generalized amplitude channel, the entanglement of different classes of two qubit systems is stable and fixed for larger interval of the channel strength.

It is well known that, quantum teleportation is one of the most important applications of entanglement. Since the first quantum teleportation protocol proposed by Bennett et. al [13], to teleport a single qubit using an entangled qubit system has been introduced, there are several versions have been suggested (see for example [14, 15, 16]). The possibility of teleporting an unknown qubit using tripartite GHZ state is discussed by Karlsson and Bourennane [17]. Gorbachev and Turbilko have introduced a teleportation protocol to teleport a two-qubit state by using GHZ state [18]. Another class of tripartite entangled state called W-state has been used to teleportate an unknown state probabilistically [19]. In 2009, Yang et. al have introduced a quantum teleportation scheme to teleport an unknown single qubit by using a different class of GHZ called GHZ-like state [20].

In this work, we investigate the effect of the generalized amplitude damping channel on two classes of tripartite entangled states: GHZ and GHZ-like states. In this study, we try to answer the following questions: (i) Is the effect of the generalized amplitude damping channel on a tripartite states similar to that predicted for two qubit systems as shown in [4, 5]? In other words, can one freeze the effect of the generalized amplitude damping channel when tripartite state passes through it. (ii) Can the fidelity of the teleported state by using these decohered tripartite states can be improved due to the local interaction as predicted for two qubit systems [8, 9]? (iii) Which state is more robust against this type of noisy channel; GHZ or GHZ-like state?

The paper is organized as follows. In Sec. 2, a description of the suggested model is introduced. The behavior of entanglement is discussed in Sec. 3. The possibility of using the decohered entangled states as quantum channels to perform quantum teleportation is studied in Sec. 4. Finally, conclusions are drawn in Sec. 5.

## 2 The system

It is assumed that a source generates states of three qubits in the GHZ or GHZ-like forms as:

$$|\psi^{(ini)}\rangle = \left\{ \begin{array}{l} |\psi_g\rangle = \frac{1}{\sqrt{2}}(\alpha|000\rangle + \beta|111\rangle), \\ |\psi_{gl}\rangle = \frac{1}{2}(c_1|001\rangle + c_2|1010\rangle + c_3|100\rangle + c_4|111\rangle), \end{array} \right\} \quad (1)$$

where  $\alpha^2 + \beta^2 = 1$  and  $c_1^2 + c_2^2 + c_3^2 + c_4^2 = 4$ . From these states we get the maximum entangled of the GHZ state ( $|\psi_g\rangle$ ) and the GHZ-like state ( $|\psi_{gl}\rangle$ ) by setting  $\alpha = \beta = 1$  and  $c_1 = c_2 = c_3 = c_4 = 1$  respectively. During the transition from the source to the three users Alice, Bob and Charlie, the qubits are forced to pass through a generalized amplitude damping channel, which is defined by the following Kraus operators [11]

$$\begin{aligned} \mathcal{E}_0 &= \frac{\sqrt{p}}{2} \left\{ (1 + \sqrt{1-\gamma}) + (1 - \sqrt{1-\gamma})\sigma_z^{(i)} \right\}, & \mathcal{E}_1 &= \sqrt{p} \left\{ \sigma_x^{(i)} + i\sigma_y^{(i)} \right\}, \\ \mathcal{E}_2 &= \frac{\sqrt{1-p}}{2} \left\{ (1 + \sqrt{1-\gamma}) - (1 - \sqrt{1-\gamma})\sigma_z^{(i)} \right\}, & \mathcal{E}_3 &= \sqrt{1-p}\sqrt{\gamma} \left\{ \sigma_x^{(i)} - \sigma_y^{(i)} \right\} \end{aligned} \quad (2)$$

where  $\sigma_j^{(i)}$ ,  $j = x, y, z$  and  $i = 1, 2, 3$  are the Pauli operators for the three qubits, respectively,  $p$  and  $\gamma$  are the strength and damping parameters of the channel. If we assume that all the three qubits are passing through this noise channel, then the final state can be written as:

$$\rho_k^{(f)} = \sum_{n=0}^{n=3} \mathcal{E}_n \rho_k^{(ini)} \mathcal{E}_n^\dagger, \quad (3)$$

where  $\rho_k^{(ini)} = |\psi_k^{(ini)}\rangle\langle\psi_k^{(ini)}|$  is the initial state of the travelling state through the generalized amplitude damping channel,  $k = g$  or  $gl$  and  $\rho_k^{(f)}$  is the final state.

In this subsection, we find the final state of travelling qubits in the noise channel (2) analytically. If we assume that, the system is initially prepared in the GHZ state as defined in Eq.(1), then by using Eq.(2), the final state (3) can be written explicitly as by

$$\rho_g^{(f)} = \mathcal{A}_1|000\rangle\langle 000| + \mathcal{A}_2|000\rangle\langle 111| + \mathcal{A}_3|111\rangle\langle 000| + \mathcal{A}_4|111\rangle\langle 111|, \quad (4)$$

where,

$$\begin{aligned}\mathcal{A}_1 &= \frac{\alpha^2}{2} \left( p^2 + (1-p)^3(1-\gamma)^3 \right), \quad \mathcal{A}_2 = \frac{1}{2} (1-\gamma)^{3/2} \left( p^2 \alpha \beta^* + \alpha^* \beta (1-p)^3 \right), \\ \mathcal{A}_3 &= \frac{1}{2} (1-\gamma)^{3/2} \left( p^2 \alpha^* \beta + \alpha \beta^* (1-p)^3 \right), \quad \mathcal{A}_4 = \frac{\beta^2}{2} \left( p^2 (1-\gamma)^3 + (1-p)^3 \right),\end{aligned}\quad (5)$$

Similarly, if the travelling state is initially prepared in the GHZ-like state  $\rho_{g\ell}$  and passes through the generalized amplitude damping channel (2), then the final state  $\rho_{g\ell}^{(f)}$  is given by

$$\begin{aligned}\rho_{g\ell}^{(f)} &= |001\rangle \left\{ \mathcal{B}_1 \langle 001| + \mathcal{B}_2 \langle 010| + \mathcal{B}_3 \langle 100| + \mathcal{B}_4 \langle 111| \right\} \\ &\quad + |010\rangle \left\{ \mathcal{B}_5 \langle 001| + \mathcal{B}_6 \langle 010| + \mathcal{B}_7 \langle 100| + \mathcal{B}_8 \langle 111| \right\} \\ &\quad + |100\rangle \left\{ \mathcal{B}_9 \langle 001| + \mathcal{B}_{10} \langle 010| + \mathcal{B}_{11} \langle 100| + \mathcal{B}_{12} \langle 111| \right\} \\ &\quad + |111\rangle \left\{ \mathcal{B}_{13} \langle 001| + \mathcal{B}_{14} \langle 010| + \mathcal{B}_{15} \langle 100| + \mathcal{B}_{16} \langle 111| \right\},\end{aligned}\quad (6)$$

where,

$$\begin{aligned}\mathcal{B}_1 &= |c_3|^2 \kappa_1, \quad \mathcal{B}_2 = c_3 c_1^* \kappa_1, \quad \mathcal{B}_3 = c_3 c_2^* \kappa_1, \quad \mathcal{B}_4 = c_3 c_4^* \kappa_2 + \frac{p^3}{4} c_4 c_3^*, \\ \mathcal{B}_5 &= c_1 c_3^* \kappa_1, \quad \mathcal{B}_6 = |c_1|^2 \kappa_1, \quad \mathcal{B}_7 = c_1 c_2^* \kappa_1, \quad \mathcal{B}_8 = c_1 c_4^* \kappa_2, \\ \mathcal{B}_9 &= c_2 c_3^* \kappa_1, \quad \mathcal{B}_{10} = c_2 c_1^* \kappa_1, \quad \mathcal{B}_{11} = |c_2|^2 \kappa_1, \quad \mathcal{B}_{12} = c_2 c_4^* \kappa_2, \\ \mathcal{B}_{13} &= c_4 c_1^* \kappa_2, \quad \mathcal{B}_{14} = c_4 c_1^* \kappa_2, \quad \mathcal{B}_{15} = c_4 c_2^* \kappa_2, \quad \mathcal{B}_{16} = |c_4|^2 \kappa_3,\end{aligned}\quad (7)$$

with,

$$\begin{aligned}\kappa_1 &= \frac{1-\gamma}{4} (p\sqrt{p} + (1-p)^{3/2}), \\ \kappa_2 &= \frac{1-\gamma}{4} (p\sqrt{p}(1-\gamma) + (1-p)^{3/2}), \\ \kappa_3 &= \frac{1-\gamma}{4} (p\sqrt{p}(1-\gamma)^3 + (1-p)^{3/2}).\end{aligned}\quad (8)$$

Since the final states of GHZ and GHZ-like states have been obtained, we can quantify the survival amount of entanglement. Also, the possibility of using these decohered states as quantum channel to perform quantum teleportation will be discussed in Sec. (4).

### 3 Entanglement

In this section, we quantify the survival amount of entanglement which is contained in the travelling state through the noisy channel. In this context, we use the tripartite negativity as a measure of entanglement. This measure states that, if  $\rho_{abc}$  represents a tripartite state, then the negativity is defined as,

$$\mathcal{N}(\rho_{abc}) = (\mathcal{N}_{a-bc} \mathcal{N}_{b-ac} \mathcal{N}_{c-ab})^{\frac{1}{3}}, \quad (9)$$

where  $\mathcal{N}_{i-jk} = -2 \sum_{\ell} \lambda_{\ell}(\rho_{ijk}^{T_i})$ ,  $\lambda_{\ell}$  are the negative eigenvalues of the partial transpose of the state  $\rho_{ijk}$  with respect to the qubit "i" [21].

In Fig.(1), the survival amount of entanglement between the three qubits who initially share a GHZ or GHZ-like state is displayed. In Fig.(1a), the entanglement behavior of

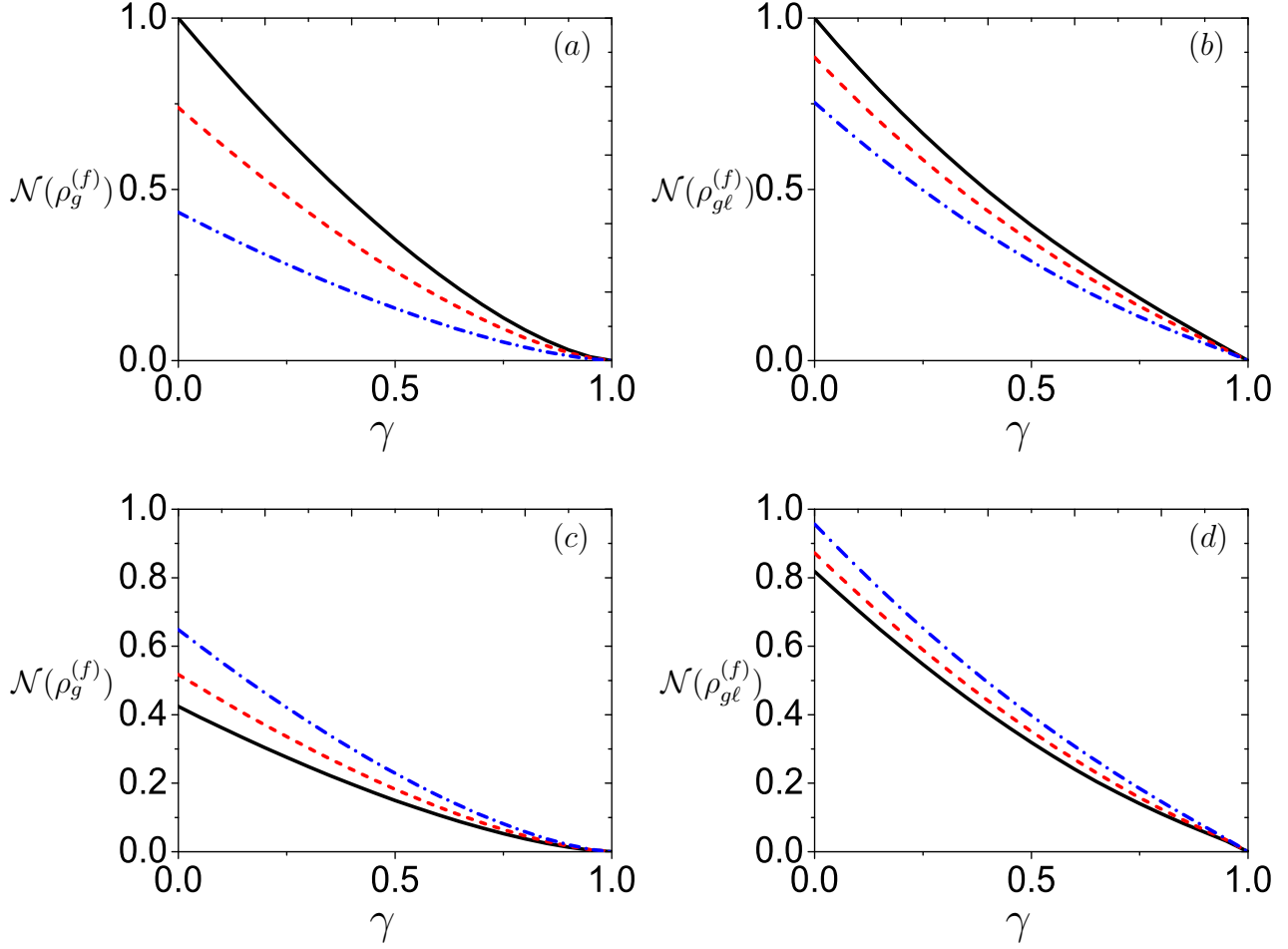


Figure 1: The entanglement  $\mathcal{N}_g(\rho^{(f)})$  (Figs.(a, c)) and  $\mathcal{N}_{gl}(\rho^{(f)})$  (Figs.(b, d)) for system initially prepared in GHZ and GHZ-like state, respectively. For figures (a,b),  $p = 0, 0.1$  and  $0.3$  for the solid, dash, and dash-dot curves, respectively, while for figures (c,d),  $p = 0.6, 0.7$  and  $0.8$  for the solid, dash, and dash-dot curves, respectively.

a system initially prepared in GHZ state with  $\alpha = \beta = 1$  passing through a generalized amplitude damping channel, is shown. The general behavior shows that,  $\mathcal{N}_g(\rho^{(f)})$  decays as  $\gamma$  increases to vanish completely at  $\gamma = 1$ . On the other hand, as the strength of the channel  $p$  increases, the upper bounds of entanglement decrease. In Fig.(1b), it is assumed that GHZ-like state is initially prepared with  $c_i = 1, i = 1, 2, 3, 4$ . It is clear that, for  $p = \gamma = 0$ , the entanglement is maximum i.e.,  $\mathcal{N}_{gl}(\rho^{(f)}) = 1$ . However, as  $\gamma$  increases further, the entanglement decays gradually to vanish completely at  $\gamma = 1$ . For small values of the channel strength, the entanglement decays gradually, while the rate of decay increases as one increases the channel strength  $p \in [0, 0.5]$ .

The behavior of entanglement for larger values of the channel's strength,  $p$  is displayed in Figs.(1c) for GHZ and (1d) for GHZ-like state. The behavior of Entanglement is similar to that depicted in Figs.(1a) and (1b), namely the entanglement decays as  $\gamma$  increases. However as one increases the channel strength  $p$ , the entanglement increases and the decay rate decreases. Comparing these two figures, one can see that the decay rate for the GHZ state is larger than that displayed for GHZ-like state.

In reality, it is difficult to keep the generated maximum entangled state isolated from

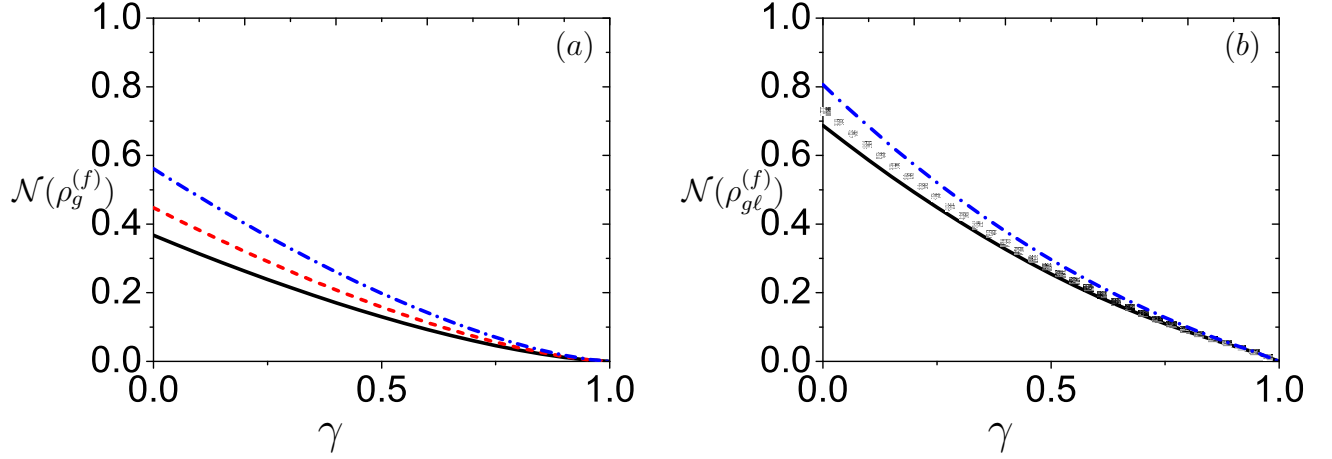


Figure 2: The solid, dash and dash-dot curves, represent the entanglement for  $p = 0.6, 0.7$  and  $0.8$  respectively, where the systems are prepared initially in (a) GHZ state with  $\alpha = 0.5$  and  $\beta = \sqrt{3}/2$ , and (b) GHZ-like state with  $c_1 = 0.8, c_2 = 0.7, c_3 = 0.6$

its surroundings and consequently it turns into partial entangled states. Therefore, it is important to investigate the behavior of these partial entangled states in the presence of this noise. For this aim, we consider Fig.(2), where it is assumed that, the users share non-maximally entangled states. Fig.(2a), displays the decohered GHZ state which is initially prepared with  $\alpha = 0.5$  and  $\beta = \sqrt{3}/2$ . The general behavior is similar to that depicted in Fig.(1) i.e., the entanglement decays as  $\gamma$  increases. However, the upper bounds of entanglements are always smaller than those shown for maximum entangled state (see Fig.(1c)). In Fig.(2a), we consider that the users share a non-maximum entangled GHZ-like state defined by  $c_1 = 0.8, c_2 = 0.7$  and  $c_3 = 0.6$ , where we consider a larger values of the channel strength. The general behavior is similar to that predicated in Fig.(1), but as  $p$  increases the decay rate decreases and consequently the upper bounds of entanglement are larger.

## 4 Teleportation

In this section, we investigate the effect of the generalized amplitude damping channel on the fidelity of the teleported state. We consider the decohered GHZ and GHZ-like states as quantum channel between the three users to perform the teleportation protocol. In this current investigation, we assume that the three users co-operate to achieve this protocol. Assume that Alice is given unknown information coded in the single qubit  $\rho_u = |\psi_u\rangle\langle\psi_u|$ , where  $|\psi_u\rangle = \mu|0\rangle + \nu|1\rangle$ ,  $|\mu|^2 + |\nu|^2 = 1$ . The initial system between the three users is given by  $\rho_s = \rho_u \otimes \rho_g^{(f)}$  or  $\rho_s = \rho_u \otimes \rho_{gl}^{(f)}$ . Now, Alice has two qubits: her own qubit and the unknown qubit, while Bob and Charlie have the second and the third qubits respectively. To perform the teleportation protocol, the users follow the following steps:

1. Alice performs Bell measurements (BM), i.e.  $\rho_{\phi^\pm} = |\phi^\pm\rangle\langle\phi^\pm|$ ,  $\rho_{\psi^\pm} = |\psi^\pm\rangle\langle\psi^\pm|$ ,  $|\phi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$ ,  $|\psi^\pm\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$  on the first two qubits; (here qubit and the unknown qubit, and Charlie makes her measurements on the basis either "0" or "1").
2. Alice and Charlie send their measures to Bob, who will do the appreciated operations

Alice	Chirile	Bob	Fidelity
$\rho_{\phi^+}$	$ x_1\rangle$	$I$	$\mathcal{F}_{g_{\phi^+}}^{(x_1)}$
	$ x_2\rangle$	$S_z$	$\mathcal{F}_{g_{\phi^+}}^{(x_2)}$
$\rho_{\phi^-}$	$ x_1\rangle$	$S_z$	$\mathcal{F}_{g_{\phi^-}}^{(x_1)} = \mathcal{F}_{g_{\phi^+}}^{(x_1)}$
	$ x_2\rangle$	$I$	$\mathcal{F}_{g_{\phi^-}}^{(x_2)} = \mathcal{F}_{g_{\phi^+}}^{(x_2)}$
$\rho_{\psi^+}$	$ x_1\rangle$	$I$	$\mathcal{F}_{g_{\psi^+}}^{(x_1)} = \mathcal{F}_{g_{\phi^+}}^{(x_1)}$
	$ x_2\rangle$	$S_z$	$\mathcal{F}_{g_{\psi^+}}^{(x_2)} = \mathcal{F}_{g_{\phi^+}}^{(x_2)}$
$\rho_{\psi^-}$	$ x_1\rangle$	$S_z$	$\mathcal{F}_{g_{\psi^-}}^{(x_1)} = \mathcal{F}_{g_{\phi^+}}^{(x_1)}$
	$ x_2\rangle$	$I$	$\mathcal{F}_{g_{\psi^-}}^{(x_2)} = \mathcal{F}_{g_{\phi^+}}^{(x_2)}$

Table 1: Teleportation protocol via decohered GHZ state as quantum channel

to get the decohered teleported information.

#### 4.1 Decohered GHZ state as quantum channel

Let us first, assume that the initial state of the system is  $\rho_s = \rho_u \otimes \rho_g^{(f)}$ , i.e., the users will use the decohered GHZ state as quantum channel. Assume that, Alice performs here measurements by using Bell state analyzers on qubits "u" and "1" [17]. If the Bell state analyzer gives  $|\phi^+\rangle_{u1}$ , or  $|\phi^-\rangle_{u1}$  then, the state of the Charlie and Bob will be projected into

$$\rho_{CB}^{\phi^\pm} = \kappa_{00}|00\rangle\langle 00| \pm \kappa_{01}|00\rangle\langle 11| \pm \kappa_{10}|11\rangle\langle 00| + \kappa_{11}|11\rangle\langle 11|, \quad (10)$$

where,

$$\begin{aligned} \kappa_{00} &= A_1(\mu^2 + \mu^*\nu + \mu^*\nu^* + \nu^2), & \kappa_{01} &= A_2(\mu^2 - \mu^*\nu + \mu^*\nu^* - \nu^2), \\ \kappa_{10} &= A_3(\mu^2 + \mu^*\nu - \mu^*\nu^* - \nu^2), & \kappa_{11} &= A_4(\mu^2 - \mu^*\nu - \mu^*\nu^* + \nu^2). \end{aligned} \quad (11)$$

To complete the protocol, Charlie, uses a spin-state analyzer with two outcomes  $|x_1\rangle = \frac{1}{2}(\sin\theta|1\rangle + \cos\theta|0\rangle)$  and  $|x_2\rangle = \frac{1}{2}(\cos\theta|1\rangle - \sin\theta|0\rangle)$ , where  $\theta$  is the analyzer angle, to measure her qubit [17]. However, if Charlie measures  $|x_1\rangle$ , or  $|x_2\rangle$  then Bob will get the final state with a fidelity given by,

$$\begin{aligned} \mathcal{F}_{g_{\phi^+}}^{(x_1)} &= \mu^2\kappa_{00}\cos^2\theta + \frac{1}{2}\sin 2\theta(\mu\nu^*\kappa_{01} + \mu^*\nu\kappa_{10}) + \nu^2\kappa_{11}\sin^2\theta, \\ \mathcal{F}_{g_{\phi^+}}^{(x_2)} &= \mu^2\kappa_{00}\sin^2\theta + \frac{1}{2}\sin 2\theta(\mu\nu^*\kappa_{01} + \mu^*\nu\kappa_{10}) + \nu^2\kappa_{11}\cos^2\theta. \end{aligned} \quad (12)$$

On the other hand, if the Bell analyzer read out is  $|\psi\rangle_{u1}^{(\pm)}$ , then the state between Charlie and Bob is projected into,

$$\rho_{CB}^{\psi^\pm} = \kappa_{11}|00\rangle\langle 00| \pm \kappa_{10}|00\rangle\langle 11| \pm \kappa_{01}|11\rangle\langle 00| + \kappa_{00}|11\rangle\langle 11|, \quad (13)$$

where  $\kappa_{ij}$ ,  $ij = 00, 01, 10$  and  $11$  are given by Eq.(11). The details of measurements and operations which can be done by the users are shown in Table (1).

The behavior of the fidelity  $\mathcal{F}_{g_{\phi^+}}^{(x_1)}$  of the teleported state by using the decohered GHZ-state (4) as quantum channel is shown in Fig.(3), where different values of the analyzer angle are considered within and without the channel strength,  $p$ . As it is described in Fig.(3a), the

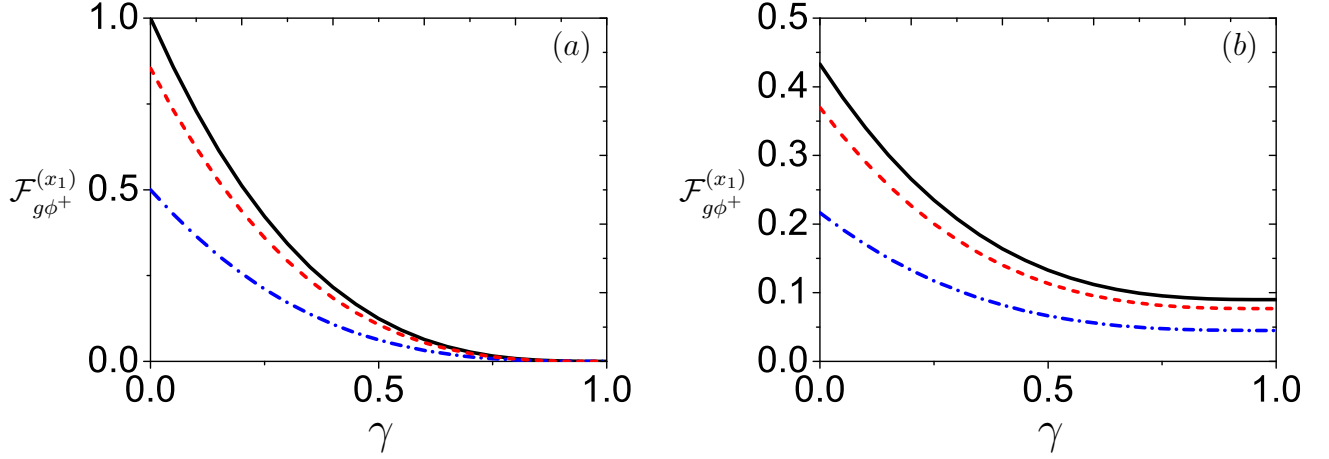


Figure 3: The fidelity of the teleported state, with the channel strength, showing solid, dash and dash-dot curves for  $\theta = 0, \pi/8$  and  $\pi/4$ , respectively for (a)  $p = 0$  and (b)  $p = 0.3$ .

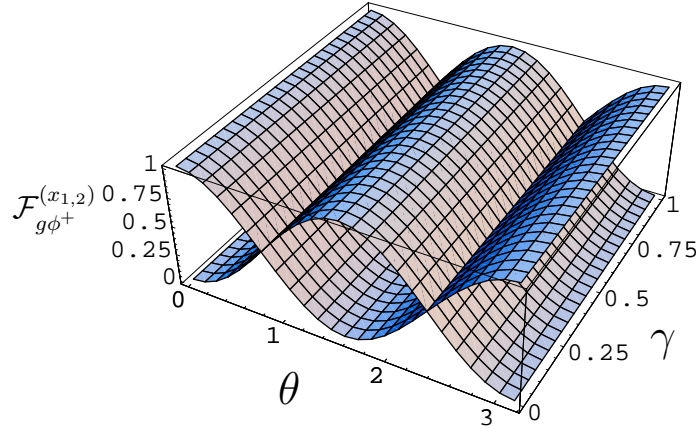


Figure 4: The fidelities  $\mathcal{F}_{g\phi^+}^{(x_{1,2})}$  of the teleported state with  $\mu = \nu = 1/\sqrt{2}$ , where the channel's strength  $p = 0$ .

fidelity decreases as the channel damping parameter  $\gamma$  increases. For small values of analyzer angle ( $\theta = 0$ ) and  $\gamma = 0$ , the initial fidelity is maximum ( $\mathcal{F}_{g\phi^+}^{(x_1)} = 1$ ). However, for larger values of the channel damping parameter, the fidelity decays smoothly to vanish completely at  $\gamma = 1$ . The initial fidelity decreases as one increases the analyzer angle as depicted for  $\theta = \pi/8$  and  $\pi/4$ , respectively. Fig.(3b) displays the effect of the channel strength  $p$  on the fidelity of the teleported state for different values of analyzer angle, where we set  $p = 0.3$ . It is clear that, the initial fidelity i.e., at  $\gamma = 0$ , is very small comparing to that displayed in Fig.(3a). On the other hand, the decay rate of the fidelity is smaller than that depicted for zero value of the channel's strength (see Fig.(3a)). Moreover, the long-lived fidelity is depicted for larger values of the channel damping parameter ( $\gamma > 0.5$ ), where the fidelity is almost constant.

Fig.(4), shows the behavior of the fidelities  $\mathcal{F}_{g\phi^+}^{(x_1)}$  and  $\mathcal{F}_{g\phi^+}^{(x_2)}$ , namely when Charlie measures  $|x_1\rangle$  and  $|x_2\rangle$ , respectively as functions in the analyzer angle  $\theta$ . It is clear that, at  $\gamma = 0$ ,  $\mathcal{F}_{g\phi^+}^{(x_1)}$  is maximum, while  $\mathcal{F}_{g\phi^+}^{(x_2)} = 0$  i.e., is minimum. As  $\theta$  increases further, the

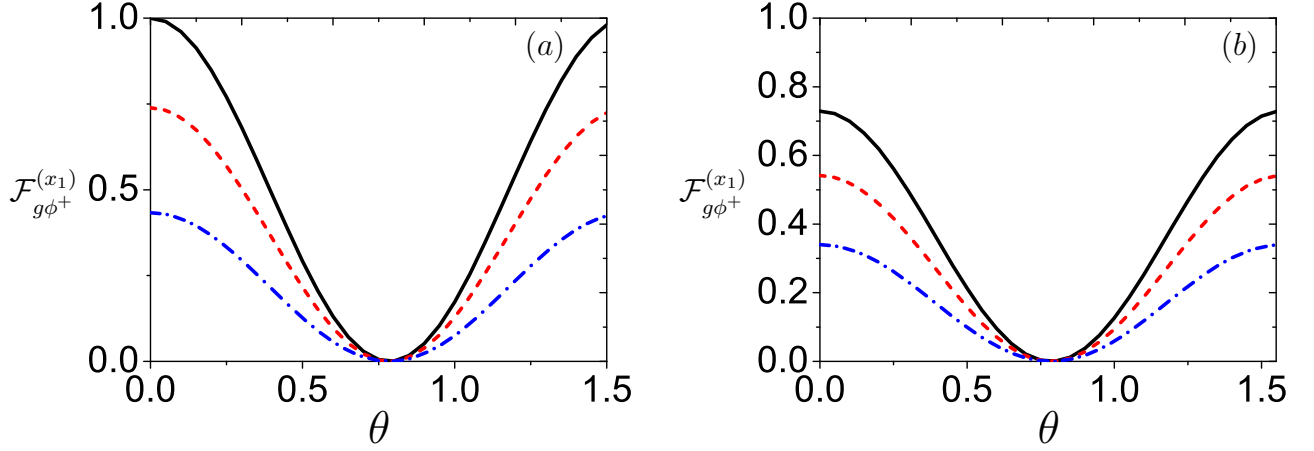


Figure 5: The fidelity of the teleported state as a function of  $\theta$ . The solid, dash and dash-dot curves for  $p = 0, 0.1, 0.3$ , respectively for (a)  $\gamma = 0$  and (b)  $\gamma = 0.1$ .

fidelity,  $\mathcal{F}_{g\phi^+}^{(x_1)}$  decreases while  $\mathcal{F}_{g\phi^+}^{(x_2)}$  increases. However at  $\theta = \pi/2$ , the situation of the behavior of the two fidelities is changing. This figure shows that, one can use the analyzer angle as a control parameter, where if Charlie decides to measure  $|x_1\rangle$  or  $|x_2\rangle$ , then by adjusting the analyzer angle, the final state can be obtained with a maximum fidelity.

In Fig.(5), we investigate the behavior of the fidelity as a function of the analyzer angle,  $\theta$  for different values of the channel strength. Fig.(5a) is devoted to study the behavior of  $\mathcal{F}_{g\phi^+}^{(x_1)}$  with zero damping channel i.e., we set  $\gamma = 0$ . As shown in Fig.(3a), the initial fidelity decreases as the analyzer angle  $\theta$  increases. It is clear that, the fidelity decreases to vanish completely at  $\theta = \pi/2$  and increases again to reach its maximum value at  $\theta = \pi$ . The upper bound of the fidelity of the teleported state decreases as the analyzer angle increases which is in agreement with the work of Karlsson and Bourennane [17]. As one increases the value of the channel damping parameter  $\gamma$ , the upper bounds of the fidelity decreases as shown in Fig.(4b), where we set  $\gamma = 0.1$ .

## 4.2 GHZ-like state as quantum channel

Now, we assume that the users share a decohered GHZ-like state (6), this means that the system is given by  $\rho_s = \rho_u \otimes \rho_{gl}^{(f)}$ . They use the same protocol described above. However depending on the results of Alice and Charlie, Bob performs the adequate operation to get the teleported state. For example, If Alice measures  $\rho_{\phi^+}$  and Charlie measures "1", then Bob will do nothing and the fidelity of the teleported state is given by  $\mathcal{F}_{gl\phi^+}^{(1)}$  as shown in Table (2).

The fidelity of the teleported state by using a decohered GHZ-like state as quantum channel is shown in Fig.(6). In Fig.(6a), we set the channel strength  $p = 0$  and assume that Alice measures  $\rho_{\phi^+}$ , while Charlie measures "0" or "1". The general behavior shows that the fidelity decays as the damping parameter  $\gamma$  increases. However, the decay rate of the fidelity depends on the Charlie's measurements. For example, if Charlie measures "1", then the rate decay of the fidelity  $\mathcal{F}_{gl\phi^+}^{(1)}$  is much smaller than that depicted for  $\mathcal{F}_{gl\phi^+}^{(0)}$ , where Charlie measures "0". Moreover, the fidelity  $\mathcal{F}_{gl\phi^+}^{(0)}$  vanishes completely at  $\gamma = 1$ , while  $\mathcal{F}_{gl\phi^+}^{(1)}$  doesn't.



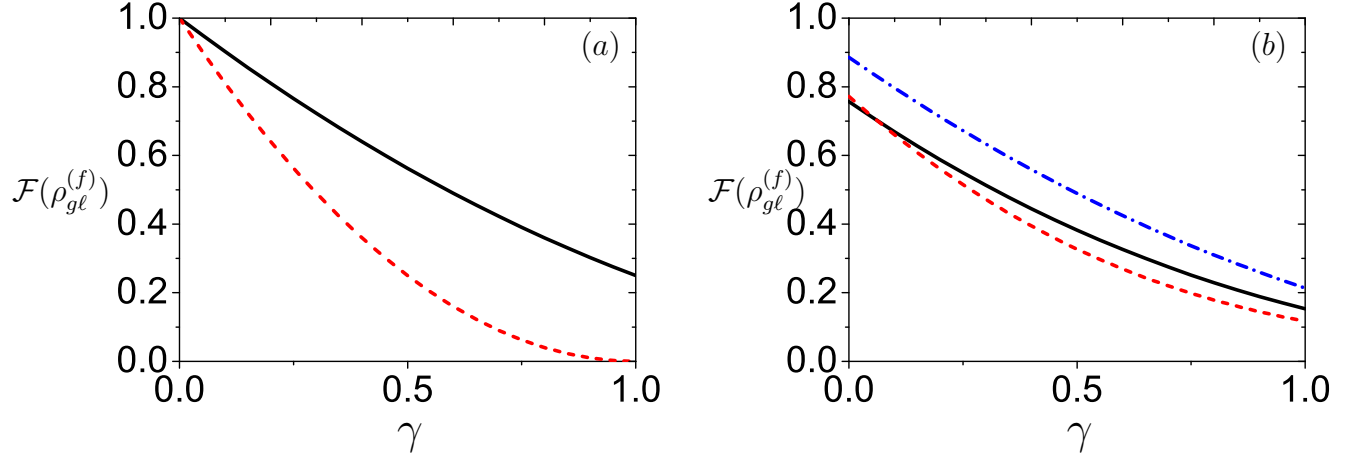


Figure 6: (a) The fidelity of the teleported state  $\mathcal{F}_{g\ell\phi^+}^{(1)}$  (solid curves) and  $\mathcal{F}_{g\ell\phi^+}^{(0)}$  (dash-curves), where the channel strength,  $p = 0.0$ . (b) The fidelity,  $\mathcal{F}_{g\ell\phi^+}^{(1)}$  for  $p = 0.1, 0.3$  and  $0.6$  for the solid, dash and dash-dot curves, respectively.

Alice	Chirile	Bob	Fidelity
$\rho_{\phi^+}$	1	$I$	$\mathcal{F}_{g\ell\phi^+}^{(1)} = \frac{1}{2}(\mu^4\mathcal{B}_1 + \mu^2\nu^2(\mathcal{B}_4 + \mathcal{B}_{13}) + \nu^4\mathcal{B}_{16})$
	0	$S_x$	$\mathcal{F}_{g\ell\phi^+}^{(0)} = \frac{1}{2}(\mu^4\mathcal{B}_6 + \mu^2\nu^2(\mathcal{B}_7 + \mathcal{B}_{10}) + \nu^4\mathcal{B}_{11})$
$\rho_{\phi^-}$	1	$S_z$	$\mathcal{F}_{g\ell\phi^-}^{(1)} = \mathcal{F}_{g\ell\phi^+}^{(1)}$
	0	$S_x S_z$	$\mathcal{F}_{g\ell\phi^-}^{(0)} = \mathcal{F}_{g\ell\phi^+}^{(0)}$
$\rho_{\psi^+}$	1	$S_x$	$\mathcal{F}_{g\ell\psi^+}^{(1)} = \frac{1}{2}(\mu^4\mathcal{B}_{16} + \mu^2\nu^2(\mathcal{B}_4 + \mathcal{B}_{13}) + \nu^4\mathcal{B}_1)$
	0	$I$	$\mathcal{F}_{g\ell\psi^+}^{(0)} = \frac{1}{2}(\mu^4\mathcal{B}_{11} + \mu^2\nu^2(\mathcal{B}_7 + \mathcal{B}_{10}) + \nu^4\mathcal{B}_6)$
$\rho_{\psi^-}$	1	$S_x S_z$	$\mathcal{F}_{g\ell\psi^-}^{(1)} = \mathcal{F}_{g\ell\psi^+}^{(1)}$
	0	$S_z$	$\mathcal{F}_{g\ell\psi^-}^{(0)} = \mathcal{F}_{g\ell\psi^+}^{(0)}$

Table 2: Teleportation protocol via decohered GHZ-like state as quantum channel

Fig.(6b) shows the behavior of the fidelity  $\mathcal{F}_{g\ell\phi^+}^{(1)}$  for different values of the channel strength  $p$ . In general, the fidelity decreases as  $\gamma$  increases. However, for small values of  $p \in [0, 5]$ , the fidelity decreases as  $p$  increases. For  $p > 5$ , the upper values of the fidelity is larger than that depicted for small values of  $p$ .

## 5 Conclusion

In this paper, we discussed the entanglement behavior of two classes of tripartite entangled states, namely, GHZ and GHZ-like states, passing through a generalized amplitude damping channel. The effect of the channel strength and channel damping parameter on the survival amount of entanglement are investigated. The decay rate of entanglement increases as the channel damping parameter increases. However, the entanglement decreases faster for small values of the channel strength, while for larger values the upper bounds of entanglement are much larger. It is shown that, the behavior of entanglement of the GHZ-like state is more robust than GHZ state, where the decay rate of entanglement for GHZ-like state is smaller

than that depicted for GHZ state.

The decohered entangled tripartite states are used as quantum channel to perform quantum teleportation. The fidelity of the teleported state is investigated within and without the channel strength. If the users use the decohered GHZ state as quantum channel, the fidelity of the teleported state depends on the channel's strength, channel's damping parameter and the analyzer's angle. It is shown that, the fidelity of the teleported state decrease quickly as the analyzer angle increases. The decay rate of the fidelity increases as the channel damping parameter increases. Within larger values of the channel strength, the decay rate increases and the fidelity decays faster. However, as the channel damping parameter increases, the behavior of the fidelity is stable and fixed.

The possibility of using decohered GHZ-like state as quantum teleportation is investigated. We show that the fidelity of the teleported state depends on Bell measurements, Von Neumann, and the channel's parameters. It is shown that, for some of the Bell measurements, the fidelity of the teleported state decays faster and completely vanishes for larger values of the channel damping, while decays slowly for others and doesn't vanish as the channel damping parameters increases. However, the decay rate of the fidelity increases as the channel strength increases. This decay can be decreased by increasing the channel's strength.

*In conclusion:* Despit the generalized amplitude damping channel causes a degradation of the entangled properties of the initial entangled states, and consequently their efficiency to perform teleportation. The upper bounds of entanglement and the fidelity of the teleported states can be enhanced as one increases the channel's strength. The phenomena of channel frozen doesn't appear for tripartite state. However, for the fidelity of the teleported state the frozen phenomena appears for larger values of the channel strength. Finally, one can say that the class of the GHZ-like state is more robust than GHZ states.

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